Worksheet Part 1: Lissajous Figures

The 3d pendulum follows trajectories given by this set of equations:

$x(t) = A * \sin(\omega_1 * t + \phi)$

$y(t) = A * \sin(\omega_2 * t)$

Where ω_1 and ω_2 are the angular frequencies of the pendulum in each direction, t is time, and ϕ is the phase difference between the swings in the two directions. Don't worry to much about phase for now, we will focus on frequency. As a reminder, $\omega = \sqrt{\frac{g}{l}}$, and this relates to the period as $T = \frac{2\pi}{\omega}$.

Since A just serves to scale the size of the trajectory (do you see why?) but doesn't change its shape, you can set A = 1 for the entire worksheet.

For part 1 of this worksheet, go to https://www.geogebra.org/m/yJNhQMQa

1. Enter the equations for x(t) and y(t). Set ϕ=0 and $\omega_1 = 1 = \omega_2$. Animate the graph and draw the resulting shape below.

2. Now keep $\omega_1 = \omega_2$ but try other values. Do you still get the same shape?

Now try the same thing but with $\omega_1 = 3$ and $\omega_2 = 4$ and draw the result.

Try other values for ω_2 & ω_1 but still keeping the ratio $\frac{\omega_2}{\omega_1} \propto \frac{3}{4}$. Does the figure change?

What about if you choose a different ratio?

By plugging in the appropriate values, graphing the trajectory and sketching the result, fill in the table below. This kind of table is called a Lissajous table, and the figures you can make with these trajectories are called Lissajous figures.

If you don't have time to fill in all of it that's ok, prioritize the $\phi = 0$ column.

$\frac{\omega_2}{\omega_1} \propto$ ↓

	$\phi = 0$	$\phi = \frac{\pi}{4}$	$\phi = \frac{\pi}{2}$	$\phi = \frac{3\pi}{4}$	$\phi = \pi$
1					
1/2					
1/3					
2/3					
3/4					

Looking at your table above try and answer the following questions. Explain your logic. It's ok if you don't have definite answers.

How can we predict the overall shape of the figure?

Can we predict how many loops a figure has? If so how?

Does the phase affect the overall shape of the figure? Describe how it changes the figure.

Bonus question (requires being more creative in the frequencies you try): What makes a figure byte its tail (if you imagine the figure is made of string there are no lose ends)?

Worksheet Part 2: String Lengths

I want to use the demo pendulum to make the beautiful figure with a ¾ frequency ratio in the Lissajous table.

Using , $\omega = \sqrt{\frac{g}{l}}$ for both ω_2 & ω_1, find an equation for $\frac{\omega_2}{\omega_1}$ in terms of l_1 and l_2.

Using the equation you found above, what lengths (in meters) should I measure out for l_1 and l_2? Assume that the entire length from the bar the pendulum is hanging from to the bob is 1.5m.